Towards a Boolean Intensional Semantics

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1 Empirical Problem

(1) a. Amy is a left-handed/female/Canadian architect.
  b. All architects are pool-players and vice-versa.
  c. Therefore, Amy is a left-handed/female/Canadian pool-player.

(2) a. Amy is a skillful architect.
  b. All architects are pool-players and vice-versa.
  c. Therefore, Amy is a skillful pool-player.

- Contrast (1) with (2). Clearly, (1) is a valid argument and (2) is invalid. How can we account for this?
- Suppose that the meaning of *architect* is given by the set of all (contextually relevant) architects. We call this set the EXTENSION of *architect*.

(3) $\textbf{architect} = \{\textbf{John}, \textbf{Amy}, \textbf{Mary}\}$ *Note:* ***bold text*** *signifies stipulated meanings.*

- Then we can naturally characterize the meaning of (1a) by making the following stipulations:

(4) a. $\textbf{left-handed} = \{\textbf{John}, \textbf{Amy}, \textbf{Bill}\}$
  b. $\textbf{left-handed architect} = \textbf{left-handed} \cap \textbf{architect} = \{\textbf{John}, \textbf{Amy}\}$
  c.
$$\begin{aligned} \textbf{Amy is a left-handed architect} &= \textbf{Amy} \in \textbf{left-handed architect} \\ &= \textbf{Amy} \in \textbf{left-handed} \cap \textbf{architect} \\ &= \textbf{Amy} \in \{\textbf{John}, \textbf{Amy}\} \end{aligned}$$

- We can then show that (1) is a valid argument:

(5) a. (1a) implies that **Amy is a left-handed architect** is true.
  b. (1b) implies that $\textbf{architect} = \textbf{pool-player}$, since we are taking the meanings of common nouns to be the extensions of those nouns.
  c.
$$\begin{aligned} \textbf{Amy is a left-handed architect} &= \textbf{Amy} \in \textbf{left-handed architect} \\ &= \textbf{Amy} \in \textbf{left-handed} \cap \textbf{architect} \\ &= \textbf{Amy} \in \textbf{left-handed} \cap \textbf{pool-player} \\ &= \textbf{Amy is a left-handed pool-player} \end{aligned}$$

- We quickly run into problems when we take this approach with *skillful*.
  - What's the extension of *skillful*, for one?
    We can imagine the set of skillful architects, but "the set of skillful individuals" is nonsensical.
  - We can only say the following,
    where $f(x_0, x_1, \ldots, x_n)$ denotes "something which is completely determined by $x_0, x_1, \ldots$ and x_n":

(6) a. $\textbf{skillful architect} = f(\textbf{skillful}, \textbf{architect})$
  b. $\textbf{skillful pool-player} = f(\textbf{skillful}, \textbf{pool-player})$
  c. In general, $\textbf{skillful architect} \neq \textbf{skillful pool-player}$.

  - Terminology: (6a) says that **skillful architect** is a FUNCTION of **skillful** and **architect**.
  - Problem: if $\textbf{architect} = \textbf{pool-player}$, $f(\textbf{skillful}, \textbf{architect}) = f(\textbf{skillful}, \textbf{pool-player})$.

* Hence, we need **architect** $\neq$ **pool-player**, even when the (2b) happens to be true.
* We'll take **architect** to be the "sense" of the word *architect*, which we call its INTENSION.
* As long as the intensions of each nouns is distinct, this allows (6) to be true, which accounts for the invalidity of (2).

2 Defining the Intension (Sense) of a Word

2.1 Standard Possible Worlds Approach

- Possible worlds semantics provides us with a notion of NOUN INTENSION: nouns denote functions from a set of "worlds" to extensions "in those worlds".
  - Example 1: Consider the meaning of *the prime minister of Canada*. The extension of this expression, in the actual world on May 25, 2017, is Justin Trudeau; the intension is a function from possible worlds to the prime minister of Canada in those worlds.
  - Example 2: Let $W = \{w_1, w_2, w_3\}$ be a set of three possible worlds. Then the meaning of the word *architect* can be given by a function **architect** as defined in the table below.

w	w_1	w_2	w_3
$\textbf{architect}(w)$	$\{\textbf{John}, \textbf{Amy}, \textbf{Bill}\}$	$\{\textbf{Amy}, \textbf{Mary}\}$	$\{\textbf{Mary}\}$

- We can now write the following meaning for *skillful n*, where n is a noun, and thus allowing (6) to be true:

(7) $\textbf{skillful n} = f(\textbf{skillful}, \textbf{n})$

- The above account is not sufficiently explanatory. To paraphrase McConnell-Ginet's (1982) objection:
  - The above account doesn't explain *why*, when the architects and pool-players happen to be the same, the skillful pool-players could be different from the skillful architects. The conceivability of alternate situations, where i.e. the set of architects differs, is not a sufficient answer - a better answer seems instead to be rooted in differences between measuring one's "skillfulness as a architect" versus one's "skillfulness as a pool-player".

2.2 Keenan (2015): Non-Atomic Boolean Lattice for CN Denotations

- Keenan (2015) shows us one way we can give definitions for the intensions of common nouns without introducing possible worlds.
- Keenan notes that the meanings of common nouns are usually given by sets of individuals and therefore have a particular mathematical structure (they form a boolean lattice).
- We can now view common noun meanings as elements of this mathematical structure, a change of perspective which makes no difference empirically.
- However, Keenan argues that a certain property of this mathematical structure was accidental (atomicity), and by removing this requirement, we end up with a new structure (a non-atomic boolean lattice with n atoms) that has a pair-like structure.
- We can now write down the following meanings:

(8) a. $\textbf{architect} = (\sigma, \tau)$ *σ is the extension, τ the intension*

b. $\textbf{pool-player} = (\sigma, \upsilon)$ *Note that the extension matches that of architect*

c. Now we can ensure that $f(\textbf{skillful}, \textbf{architect}) \neq f(\textbf{skillful}, \textbf{pool-player})$, even though the extensions are equal (i.e. $\sigma = \sigma$), since it need not be the case that $f(\textbf{skillful}, (\sigma, \tau)) = f(\textbf{skillful}, (\sigma, \upsilon))$.

2.3 Wymark (in progress): Atomless Boolean Lattice for All Denotations

- Keenan (2015) states offhandedly that "for expressions in general," i.e. nouns, verbs, sentences, and so on, "[he] doubt[s] there is a uniform notion of intension".
- Without heeding this warning, I attempted to generalize Keenan's approach, which only modeled common nouns and their modifiers, by treating a structure similar to the one he uses (an atomless boolean lattice) as a universal domain for intensions of all categories.
- In order to simplify the task of semantic interpretation, which translates tree-like syntactic structures into more abstract logical forms, I view each element of this universal domain as a binary branching tree.
  - ⋆ This is achieved by (carefully!) defining a binary operator on the domain. It relies on a special subset of the domain whose elements are declared "irreducible", allowing them to serve as the leaves of these trees.
- In this semantics, there are no extensions – it is purely intensional. Intensions are treated as primary, which makes sense: they can be thought of as mental symbols, as opposed to extensions, which become associated with these symbols in context.
  - ⋆ In a standard semantics, extensions are crucial for establishing facts about entailment. Since the domain for intensions is a boolean lattice, we can use the partial order on the lattice to model entailment.
  - ⋆ I also show that it is possible to build a system that computes extensions from these intensions, if you so desire, but it is arguable that such a system is not empirically necessary.
- We can now write down the following meanings:

(9) **architect** $= a$ — a *is some element of the universal domain*

(10) **pool-player** $= p$ — p *is some element of the universal domain, distinct from* a

(11) Now we can ensure that $f(\textbf{skillful}, \textbf{architect}) \neq f(\textbf{skillful}, \textbf{pool-player})$, since it need not be the case that $f(\textbf{skillful}, a) = f(\textbf{skillful}, p)$.

3 Conclusion

- Three approaches to accounting for (1) and (2) have been sketched, each depending on a distinct notion of intension:
  1. The possible worlds approach builds intensions out of extensions, and in this sense makes extensions primary;
  2. Keenan's (2015) approach uses a pair-like structure, which puts intensions and extensions on an equal footing;
  3. my approach uses a "universal domain" for intensions, from which extensions could, in principle, be computed, making intensions primary.
- There are other approaches to accounting for (1) and (2), with the most appealing alternative being Larson's (1998) event semantics approach, which does not make any use of intensions. Instead, it introduces "events" into the extensions of common nouns and makes use of this new distinction to allow for the invalidity of (2).

References

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[2] Larson, R. 1998. *Events and modification in nominals.* Semantics and Linguistic Theory. Vol. 8.

[3] McConnell-Ginet, S. 1982. *Adverbs and logical form: a linguistically realistic theory.* Language: 144-184.